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"On the parity nonconservation induced by
the Universal Fermi Interactions into
the Pion-Nucleon Vertex"

D. Flamm and P.G.O. Freund

Institute for Theoretical Physics
University of Vienna
Vienna, Austria

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Abstract

The problem whether the interpretation of strong interactions as high energy effects of the Universal Fermi Interactions (UFI) is consistent with experimental evidence on parity conservation in low energy nuclear physics is investigated. The parity non conserving part of the one nucleon off shell π -N vertex, which originates in the UFI (which we consider smeared out with a heavy vector boson of such a mass, that they bind an extreme relativistic nucleon - antinucleon pair into a pion) of the nucleons, is evaluated using dispersion methods and is found to have a relative magnitude of order 10^{-5} when compared with the parity conserving part. This yields a parity nonconserving π -N scattering amplitude of the same relative order of magnitude, a result which does not contradict the existing experimental data.

1. Introduction

Since at high energies the local Universal Fermi Interactions (UFI) become strong it has been proposed ¹⁾ that they might bind a $N-\bar{N}$ pair into a pion and account for the strong π -N interaction. The UFI being parity nonconserving (PNC) one would expect that they induce a parity nonconservation also into the strong interactions. On the other hand there is good experimental evidence for parity conservation ²⁾ (the relative strength of the PNC amplitude is of order $\leq 10^{-4}$) in low energy nuclear physics. It is therefore necessary to investigate whether the PNC in strong interactions originating from UFI is compatible with the above mentioned experimental evidence.

¹⁾ K. Baumann, P.G.O. Freund and W. Thirring, Nuovo Cim. 18, 906 (1960) and B. Juvet (private communication to prof. Thirring)

²⁾ D.E. Wilkinson, Phys. Rev. 109, 1610 (1958)

F. Boehm and J. Hauser, Bull. Am. Phys. Soc. Ser. II 4, 460 (1959)

From the theoretical point of view the problem of PNC in strong interactions has been discussed by many authors ³⁾. It has been proved that because of charge symmetry for the π -N vertex with the two nucleons on the mass-shell, CP invariance implies P and C invariance separately. Nevertheless this is not the case if one of the nucleons is kept off-shell. Since in π -N scattering such a vertex plays an important role, it is interesting to get an idea about ^{the} order of magnitude of its PNC part. This we shall do using dispersion theory. We shall write a dispersion relation for the π -N vertex with one nucleon off-shell and shall consider in the unitarity relation the lowest mass parity conserving (π ,N) and the lowest mass PNC (B,N) states (B being the intermediate boson of the UPI interaction). For the mass M of the

³⁾ W. Thirring, Nucl. Phys. 10, 97 (1959); 14, 565 (1960)

V.G. Soloviev, Nucl. Phys. 6, 618 (1958)

G. Morpurgo and B.F. Touschek (unpublished)

S. Fubini and D. Walecka, Phys. Rev. 116, 194 (1959)

G. Barton, Nuovo Cim. 19, 512 (1961)

intermediate boson we shall insert the value deduced in ref. 1) from the condition that the UFI be strong enough to bind an extreme relativistic $N-\bar{N}$ pair into a pion. It is this choice of the value of M that makes the whole calculation nontrivial, since in this case the UFI coupling constant $g_F M^2$ is of the order of unity and its effects could be comparable with those of strong interactions; another choice of M (say of the order of magnitude of the nucleon mass) would imply the usual weak interaction, which is evidently negligible against the strong π - N interaction. It is this strong energy dependence of the UFI upon which the philosophy of the whole program initiated in references 3) and 1) rests.

Our result is the following: the PNC admixture in the one nucleon off-shell π - N vertex induced by the (B,N) intermediate state is of the order $10^{-2} m/M \sim 10^{-5}$ (m = nucleon mass) in amplitude. This is consistent with the existent experimental evidence ²⁾ and thus shows that the model advanced in ref. 1) cannot be discarded by parity conservation arguments in low

energy nuclear physics.

2. The one nucleon off-shell π -N vertex.

We shall consider the vertex

$$\begin{aligned}
 \langle 0 | f(0) | p, s^{\text{in}}, 1 \lambda \rangle &= -ig \tilde{\gamma}_5 i\gamma q \left[\gamma_5 F_1(x) + iF_2(x) \right] \cdot \\
 &\quad \cdot i\gamma p \frac{u(p, s)}{(p_0/m)^{1/2}} \frac{\epsilon_f(\lambda)}{(2\ell_0)^{1/2}} = \\
 &= -ig \tilde{\gamma}_5 \left[\frac{x - i\gamma q}{2x} (\Gamma_1(x)\gamma_5 + i\Gamma_2(x)) + \frac{x + i\gamma q}{2x} (\Gamma_1(-x)\gamma_5 + \right. \\
 &\quad \left. + i\Gamma_2(-x)) \right] \frac{u(p, s)}{(p_0/m)^{1/2}} \frac{\epsilon_f(\lambda)}{(2\ell_0)^{1/2}}
 \end{aligned} \tag{1}$$

Here $p, s (\ell, \lambda)$ are the momentum and spin - isospin of the nucleon (π -meson); $f(x) = (i\gamma \partial + m)\psi(x)$, $\psi(x)$ being the nucleon field operator; $q = p + \ell$; $x^2 = -q^2$ (we use a $(-1, 1, 1, 1)$ metric); $\Gamma_1(\Gamma_2)$ is the form factor corresponding to the parity conserving (nonconserving) part of the vertex. The first form of the matrix element used in (1) follows directly from invariance considerations under the Lorentz-group and PC-conservation, while the second form (which we shall use in the following) is just a

convenient rewriting of the first one ⁴⁾. For Γ_i ($i = 1, 2$) we assume the correctness of the following unsubtracted dispersion relations

$$\Gamma_i(x) = 1/\pi \int_{-\infty}^{\infty} \frac{\text{Im} \Gamma_i(x') dx'}{x' - x - i\varepsilon} \quad (2)$$

Using standard techniques one can obtain expressions for the absorptive parts of Γ_i as sums over intermediate states of which we select the lowest mass parity conserving (πN) (fig.1) and PNC (BN) (fig.2) states.

The contribution from fig.1 to the absorptive part of Γ_i is, as one can easily see ⁴⁾

$$\text{Im} \Gamma_i^{\pi}(x) = h_i^*(x) \Gamma_i(x) \quad (i = 1, 2) \quad (3)$$

where

$$\begin{aligned} h_1(x) &= \sin \alpha_{11}(x) e^{i\alpha_{11}}(x) \\ h_2(x) &= \sin \alpha_1(x) e^{i\alpha_1}(x) \end{aligned} \quad (4)$$

4) The parity conserving part of the one nucleon off-shell π -N vertex has been investigated by A. Bincer, *Phys. Rev.* 118, 885 (1960)

α_1 and α_{11} being the $T = 1/2, S_{1/2}$ respectively $T = 1/2 P_{1/2}$ phase shifts of the π -N scattering.

Inserting (3), (4) into (2) one obtains two independent homogeneous integral equations of the Omnès⁵⁾ type. These can be solved in the normal way and for us it is important to observe that they admit a solution of the form

$$\begin{aligned} \Gamma_1(x) &= G_1(x) \\ \Gamma_2(x) &= 0 \end{aligned} \tag{5}$$

with

$$\begin{aligned} G_1(x) &= \exp \left[Q_1(x) - Q_1(m) \right] \\ Q_1(x) &= 1/\pi \int_{-\infty}^{\infty} \frac{\alpha_{11}(x')}{x' - x - i\epsilon} dx' \end{aligned} \tag{6}$$

The factor $\exp \left[- Q_1(m) \right]$ in $G_1(x)$ is due to the fact that by definition $\Gamma_1(m)$ is normalized to unity. The solution (5) is strictly parity conserving and it was to be expected that the completely parity conserving graph fig.1 should yield a parity

⁵⁾ R. Omnès, Nuovo Cim. 8, 316 (1958)

conserving solution. The unphysical integration region ($x < 0$)

in (6) can be avoided using the relation ⁶⁾

$$\alpha_{11}(-x) = \alpha_1(x) \quad (7)$$

Now let us see the PNC induced by the graph fig.2. The contribution

of this graph to the absorptive parts of Γ_i we compute using

first order perturbation theory for the two NN vertices and the

nucleon propagator and inserting $G_1(x)$ for the πN vertex

(marked in fig.2 by a circle) since considering the PNC part

of this πN vertex would lead to higher order effects. Hence the

contribution of graph fig.2 will not contain Γ_i and will thus

lead to an inhomogeneity in the Omnès equations. Writing down

these contributions in the above mentioned way using a V-A NN

interaction and performing the integrations in the c.m. system

one obtains after lengthy but straightforward computations

⁶⁾ G.F. Chew, M.L. Goldberger, F.E. Low, Y. Nambu, Phys. Rev.

$$\begin{aligned} \text{Im } \Gamma_2^B(x) = & G^2 \frac{\pi}{(2\pi)^3} \frac{\left[x^2 - (m+M)^2 \right]}{\mu^2 - (x \mp m)^2} \frac{1}{(m^2+a^2)^{1/2} + (M^2+a^2)^{1/2}} \\ & \frac{x}{b} \int_{V_-}^V \frac{dV}{m^2 - V^2} \left\{ \left[m(M^2+a^2)^{1/2} - m(m^2+b^2)^{1/2} \right] \mp \right. \\ & \left. \frac{M^2 - m^2 - V^2}{2} \right] E(V) + \left[m(m^2+a^2)^{1/2} \mp x(m^2+b^2)^{1/2} \right] \mp \\ & \left. \frac{V^2 - m^2 - M^2}{2} \right] F(V) \right\} \end{aligned} \quad (8)$$

where

$$\begin{aligned} a = & \left[\frac{(x^2 + m^2 - M^2)^2}{4x^2} - m^2 \right]^{1/2}; \quad b = \left[\frac{(x^2 + m^2 - \mu^2)^2}{4x^2} - m^2 \right]^{1/2} \\ V_{\pm} = & \left[\pm 2ab + m^2 + M^2 - 2(m^2+b^2)^{1/2} (M^2+a^2)^{1/2} \right]^{1/2} \\ E(V) = & mV \left[G_1^*(V) + G_1^*(-V) \right] + m^2 \left[G_1^*(V) - G_1^*(-V) \right] \\ F(V) = & mV \left[G_1^*(V) + G_1^*(-V) \right] + V^2 \left[G_1^*(V) - G_1^*(-V) \right] \end{aligned} \quad (9)$$

M, m, μ are the B, N, π masses

G is the BN coupling constant

The integral equation for Γ_2 now reads

$$\Gamma_2(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{h_2^*(x') \Gamma_2(x') dx'}{x' - x - i\epsilon} + f_2(x) \quad (10)$$

with

$$f_2(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\text{Im } \Gamma_2^B(x') dx'}{x' - x - i\epsilon} \quad (11)$$

We are interested in that solution of the inhomogeneous equation (10) which in the homogeneous case ($f_2(x) = 0$) yields the parity conserving solution (5). This solution of equation (10) is ⁵⁾

$$\begin{aligned} \Gamma_2(x) = f_2(x) \cos \alpha_1(x) + 1/\pi \exp \left[\vartheta(x) + \alpha_{11}(x) \right] \cdot \\ \cdot P \int_{-\infty}^{\infty} \frac{f_2(x') \sin \alpha_1(x') \exp \left[-\vartheta(x') \right] dx'}{x' - x - i\epsilon} \end{aligned} \quad (12)$$

where

$$\vartheta(x) = 1/\pi P \int_{-\infty}^{\infty} \frac{\alpha_1(x')}{x' - x} dx' \quad (13)$$

and again at the computation of the integrals use is to be made of the relation (7). Eqs. (8) - (13) allow the computation of

$\Gamma_2(x)$ as soon as one knows $M, m, \mu, G, \alpha_1(x), \alpha_{11}(x)$.

3. Numerical results and conclusions

From (6) - (13) one sees that Γ_2 contains integrals of $\alpha_1(x)$ and $\alpha_{11}(x)$ times certain functions of x between threshold and ∞ . The values of these phase shifts are not known in the high energy region. Nevertheless one can overcome this difficulty reasoning in the following way. At high energies the phase shifts will

have a big imaginary part (due to the predominance of inelastic effects) while their real part decreases. Since at the computation of $f_2(x)$ the integrals of the α 's appear through the G_1^* factors in $\text{Im} \Gamma_2^B$ and here the α integrals appear in the exponent, their imaginary parts produce only phase factors. An analogous reasoning is valid also for the second term in (12) where one has one more integration and a $\sin \alpha_1$ which nevertheless has a modulus ≤ 1 even at high energies. So one could take the real parts of the α_1 and α_{11} phase shifts and add the modulus of the two terms in (12). One obtains in this way an upper limit for the real value of $\Gamma_2(x)$. Inserting in the formulae of section 2

$$\alpha_{11}(x) = 0$$

$$\alpha_1(x) = (10.1x^2 - 24.6x + 14.5) \text{ (14)*} \text{ (13.2 - x) (x - 1.03)}$$

(the parabola for α_1 being chosen to fit the experimental data ⁷⁾)

*) In (14) x is dimensionless and numerically equal to the value of $\sqrt{-q^2}$ in BeV

7) E.L. Lomon, 1958 Ann. Int. Conf. on High Energy Physics at CERN

one obtains

$$\mathcal{F} = \left| \frac{\Gamma_2(x)}{\Gamma_1(x)} \right|_{x \ll M^2} \sim 10^{-2} \frac{m}{M} G^2 \quad (15)$$

for the order of PNC in amplitude near the mass shell (the region interesting in low energy nuclear physics). Since $G^2 M^{-2}$ has to equal the UFI coupling constant $g_F = 10^{-5} m^{-2}$ one can from (15) see that $\mathcal{F} \sim 10^{-7} M/m$ and thus strongly depends on M . Choosing for M the value which is necessary for the UFI to bind an extrem relativistic $N-\bar{N}$ pair into a π -meson which by ref.1) is ~ 400 BeV one finds $\mathcal{F} \leq 10^{-5}$, which does not contradict experimental evidence ²⁾. This same order of magnitude for PNC evidently appears in the Born approximation to π -N scattering amplitude due to the $\Gamma_1 \Gamma_2$ terms.

It is interesting to remark that a perturbation theory treatment of the graph fig.2 would yield $\mathcal{F} \sim \frac{1}{2\pi^2} \frac{m^2}{M^2} G^2$. This result is independent of M and is smaller than (15). This happens because for the particular case of a V-A interaction the graph fig.2 gives a convergent result so that it cannot depend on the

cutoff mass. Nevertheless higher order ^{graphs} would give divergent and thus M dependent contributions which might be bigger than the lowest order graph fig.2 and could account for the result (15) obtained by dispersion theory. At any rate first order perturbation theory cannot be relied upon in this case.

Acknowledgments

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Figure Captions

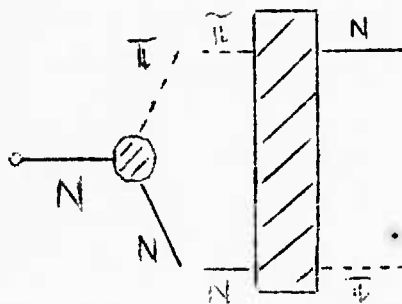


fig. 1

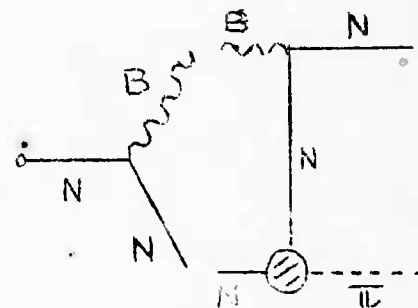


fig. 2

External lines marked with a small circle at their end represent off-shell particles.